

Morning (Time: 2 hours 30 minutes) $\quad$ Paper Reference 4MB0/02R

## Mathematics B

Paper 2R


You must have: Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided
- there may be more space than you need.


## Information

- The total mark for this paper is 100
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Without sufficient working, correct answers may be awarded no marks.



## Answer ALL ELEVEN questions.

## Write your answers in the spaces provided.

You must write down all the stages in your working.

$$
\mathbf{A}=\left(\begin{array}{ccc}
1 & 5 & -4 \\
2 & -1 & 3
\end{array}\right) \quad \mathbf{B}=\left(\begin{array}{ll}
1 & 2 \\
3 & 4 \\
5 & 6
\end{array}\right)
$$

(a) Find $\mathbf{A B}$
(b) Find BA

Question 1 continued

2 Calculate the value of $\frac{70 \times 10^{101}}{5 \times 10^{-46}}+\frac{110 \times 10^{-28}}{2 \times 10^{-177}}$
Give your answer in standard form.
You must show all your working.
$3 y$ varies inversely as the square of $x$.
(a) Write down a formula for $y$ in terms of $x$ and a constant $k$.

Given that $\quad \frac{\mathrm{d} y}{\mathrm{~d} x}=-\frac{5}{32} \quad$ when $x=4$
(b) find the values of $x$ when $y=\frac{45}{4}$

4 Given that $(x-3)$ is a factor of $5 x^{3}+2 x^{2}+k x+36$ where $k$ is a constant,
(a) show that $k=-63$

Given that, for all values of $x$,

$$
5 x^{3}+2 x^{2}-63 x+36=(x-3)\left(a x^{2}+b x+c\right)
$$

where $a, b$ and $c$ are constants,
(b) find the value of $a$, the value of $b$ and the value of $c$.

Question 4 continued

5 In 2017, a supermarket bought 20000 oranges at a cost price of US $\$ 0.65$ each. All the oranges were sold for a selling price of US $\$ 0.80$ each.
(a) Calculate the total profit, in US \$, the supermarket made by selling all the oranges.

In 2018, the supermarket bought $n$ oranges at a cost price for each orange that was $20 \%$ less than the cost price for each orange in 2017.

All the oranges were sold for a selling price for each orange that was $12.5 \%$ less than the selling price for each orange in 2017.

In 2018, the total profit the supermarket made by selling all $n$ oranges was US $\$ 4500$
(b) Find the value of $n$.

Question 5 continued

6 The triangle $A$ has vertices with coordinates $(-2,9),(-2,6)$ and $(-3,8)$.
(a) On the grid, draw and label triangle $A$.

Triangle $B$ is the image of triangle $A$ under the translation $\binom{3}{-6}$
(b) On the grid, draw and label triangle $B$.

Triangle $C$ is the image of triangle $B$ under a reflection in the line with equation $x+y=5$
(c) On the grid, draw and label triangle $C$.

The matrix $\mathbf{T}=\left(\begin{array}{rr}0 & 2 \\ -2 & 0\end{array}\right)$
Triangle $D$ is the image of triangle $C$ under the transformation with matrix $\mathbf{T}$.
(d) On the grid, draw and label triangle $D$.

The ratio (area of triangle $B):($ area of triangle $D)=1: n$, where $n$ is an integer.
(e) Find the value of $n$.

Question 6 continued


Question 6 continued

Only use this grid if you need to redraw your triangles.



Figure 1
Figure 1 shows triangle $O A B$ in which $\overrightarrow{O A}=12 \mathbf{a}$ and $\overrightarrow{O B}=8 \mathbf{b}$.
(a) Express $\overrightarrow{A B}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

The point $P$ lies on $O A$ such that $A P: A O=1: m$, where $m$ is a constant.
(b) Find $\overrightarrow{A P}$ in terms of $m$ and $\mathbf{a}$.

The point $Q$ lies on $A B$ such that $A Q: A B=1: n$, where $n$ is a constant.
(c) Find $\overrightarrow{A Q}$ in terms of $n$, a and $\mathbf{b}$.
(d) Hence find an expression for $\overrightarrow{P Q}$ in terms of $m, n$, a and $\mathbf{b}$.

Simplify your expression.
Given that $\overrightarrow{P Q}=3 \mathbf{a}+2 \mathbf{b}$
(e) find the value of $m$ and the value of $n$.

The area of triangle $O A B$ is $20 \mathrm{~cm}^{2}$
(f) Calculate the area, in $\mathrm{cm}^{2}$, of $O P Q B$.

Question 7 continued

Question 7 continued

Question 7 continued


Figure 2
Figure 2 shows a circle $A B C$ with centre $O$.
$A B$ is a chord of the circle and $M$ is the midpoint of the chord.
(a) Explain why $\angle O M A=90^{\circ}$

$$
\begin{equation*}
A B=10 \mathrm{~m} \quad \text { and } \quad \angle O A B=20^{\circ} \tag{1}
\end{equation*}
$$

(b) Calculate the length, in m to 3 significant figures, of $O A$.

The chord $B C$ is extended to the point $D$ so that $D A$ is the tangent to the circle at $A$.
Given that $\angle O D A=40^{\circ}$
(c) calculate, in m to 3 significant figures, the length of $A D$,
(d) calculate, in $m$ to 3 significant figures, the length of $B D$.
(e) Calculate the size, in degrees to 3 significant figures, of $\angle A B D$.

$$
\left[\begin{array}{cl}
\text { Sine rule: } & \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
\text { Cosine rule: } & a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{array}\right]
$$

Question 8 continued

Question 8 continued

Question 8 continued

9 Each morning, Jenny takes at random one carton of fruit juice from her fridge and drinks all the juice in the carton with her breakfast. She does not return the carton to the fridge.

Before breakfast on Monday, Jenny has in her fridge
4 cartons of orange juice ( $O$ )
8 cartons of grapefruit juice $(G)$
Jenny does not buy any more fruit juice before breakfast on Wednesday.
The incomplete probability tree diagram, on page 23 , shows information about the fruit juice that Jenny drank on Monday, on Tuesday and on Wednesday.
(a) Complete the probability tree diagram.
(b) Calculate the probability that Jenny will drink
(i) orange juice on all three mornings,
(ii) orange juice on at least one of the three mornings.

Each carton of orange juice contains 250 millilitres of juice.
Each carton of grapefruit juice contains 200 millilitres of juice.
For the three mornings, Jenny does not want to drink a total of more than 650 millilitres of fruit juice.
(c) Write down the possible combinations of fruit juice that Jenny could drink on the three mornings so that she does not drink a total of more than 650 millilitres of fruit juice.
(d) Calculate the probability that Jenny does not drink a total of more than 650 millilitres of fruit juice on the three mornings.

Question 9 continued


Question 9 continued

Question 9 continued
$10 y=\frac{9}{x^{2}}+2 x-\frac{8}{x}$
(a) Complete the table of values for $y=\frac{9}{x^{2}}+2 x-\frac{8}{x}$

Give your values of $y$ to 2 decimal places.

| $\boldsymbol{x}$ | 0.75 | 1 | 1.25 | 1.75 | 2 | 2.5 | 3 | 3.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | 6.83 | 3 |  |  | 2.25 |  | 4.33 | 5.45 |

(b) On the grid opposite, plot the points from your completed table and join them to form a smooth curve.
(c) Using your curve, write down an estimate, to 2 decimal places, of the value of $x$ for
which $\frac{9}{x^{2}}+2 x-\frac{8}{x}$ has a minimum value in the interval $0.75 \leqslant x \leqslant 3.5$
(d) Show that $4 x^{3}-6 x^{2}-8 x+9=0$ can be written in the form

$$
\frac{9}{x^{2}}+2 x-\frac{8}{x}=a x+b
$$

where $a$ and $b$ are integers.
Give the value of $a$ and the value of $b$.
(e) Hence, by drawing a suitable straight line on the grid, find estimates, to 2 decimal places, of the solutions of the equation $4 x^{3}-6 x^{2}-8 x+9=0$ in the interval $0.75 \leqslant x \leqslant 3.5$


Turn over for a spare grid if you need to redraw your curve.

Question 10 continued

Only use this grid if you need to redraw your curve.



Figure 3
Information about the function f is shown in Figure 3.
Given that f is the mapping $\mathrm{f}: x \mapsto a x^{3}+b$ where $a$ and $b$ are constants,
(a) use the information in Figure 3 to write down and simplify two equations in $a$ and $b$.
(b) Hence find the value of $a$ and the value of $b$.
(c) Using your values of $a$ and $b$, express the inverse function $\mathrm{f}^{-1}$ in the form $\mathrm{f}^{-1}: x \mapsto \ldots$
g is the function defined as $\mathrm{g}: x \mapsto \frac{p x}{x-p}$ where $x \neq p$ and $p$ is a constant.
(d) Find $\mathrm{g}(2 p)$, simplifying your answer.
(e) Show algebraically that $\operatorname{gg}(x)=x$

Show clear algebraic working.
(f) Hence write down the inverse function $\mathrm{g}^{-1}$ in the form $\mathrm{g}^{-1}(x)=\ldots$

Question 11 continued

Question 11 continued

